Lessons no. 2, 3

21 February, 1 March 2022

1. Consider two affine transformations represented by matrices

$$M_1 = \begin{pmatrix} 6 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}, \; M_2 = \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 6 \end{pmatrix}.$$

   What is the difference between the results of applying these transformation onto a model of a house?

2. Consider a point light source (lamp), which is placed in the point $S = (0, 5, 10)^\top$ and two pencils, which cast a shadow onto a plane $\rho : z = 0$. The endpoints of the first pencil are $A = (0, 5, 5)^\top$ and $B = (0, 6, 5)^\top$, and the endpoints of the second pencil are $C = (5, 5, 1)^\top$ and $D = (5, 6, 1)^\top$.

   (a) How do the shadows of the pencils look?

   (b) Find the matrix of a projection (4×4), which determines the coordinates of the shadow of a point $(x, y, z, 1)^\top$. Show the analogy with the equations of perspective projection.

   (c) Under what conditions would orthographic projection be used?

3. What is the result of cross product of vectors $\mathbf{v} = (v_1, v_2, v_3)^\top$ and $\mathbf{w} = (w_1, w_2, w_3)^\top$? Show the connection to the area of a parallelogram determined by vectors $\mathbf{v}, \mathbf{w}$.

4. Consider a triangle with vertices $V_1 = (0, 2, 2)^\top, V_2 = (0, 1, 4)^\top, V_3 = (1, 1, 2)^\top$, which are ordered counter-clockwise, while looking at its visible side. Find the normal vector of length 1, which is pointing outwards from its visible side.

5. Compute the area of a triangle determined by points $(x_1, y_1)^\top, (x_2, y_2)^\top, (x_3, y_3)^\top$. How can we determine, whether a point X lies inside the triangle?

6. Compute the area of a simple polygon determined by points $(x_1, y_1)^\top, ..., (x_n, y_n)^\top$. How can we determine, whether a point X lies inside the polygon?

7. Find the intersection of a ray $p = P + t\mathbf{u}$, where $P = (p_1, p_2, p_3)^\top$, $t \in \mathbb{R}$, $\mathbf{u} = (u_1, u_2, u_3)^\top$ and $|\mathbf{u}| = 1$, with

   (a) a plane determined by a point $P_1 = (x_1, y_1, z_1)^\top$ and a non-zero normal vector $\mathbf{n} = (a, b, c)$,

   (b) a triangle $T_1T_2T_3$,

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(c) a polygon $M_1M_2...M_n$,

(d) a sphere with a radius r and a center $C = (c_1, c_2, c_3)^\top$.
Verify your result on the sphere, where $r = 1$, $C = (1, 1, 0)^\top$ and the three rays are given by

$$l_1 = (2, 0, 0)^\top + t(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0)^\top,$$

$$l_2 = (2, 0, 0)^\top + t(0, 1, 0)^\top,$$

$$l_3 = (2, 0, 0)^\top + t(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0)^\top,$$

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