

Scanline

Practical problem step by step

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Problem

Fill the polygon $ABCDE$ using the scanline algorithm, where

$$A = \begin{pmatrix} 1 \\ 3 \end{pmatrix}, \quad B = \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \quad C = \begin{pmatrix} 7 \\ 2 \end{pmatrix}, \quad D = \begin{pmatrix} 4 \\ 2 \end{pmatrix}, \quad E = \begin{pmatrix} 5 \\ 5 \end{pmatrix}.$$

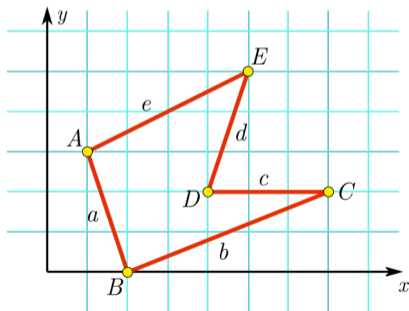
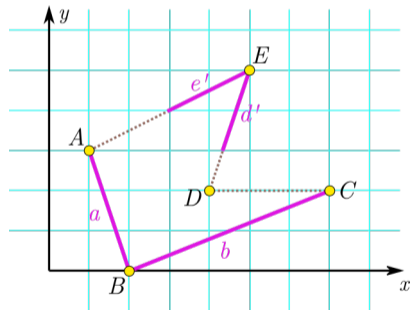


Table of edges (TE)

y	edges						
0	a <table border="1"><tr><td>3</td><td>2</td><td>$-1/3$</td></tr></table> b <table border="1"><tr><td>2</td><td>2</td><td>$5/2$</td></tr></table>	3	2	$-1/3$	2	2	$5/2$
3	2	$-1/3$					
2	2	$5/2$					
1	\emptyset						
2	\emptyset						
3	d' <table border="1"><tr><td>5</td><td>$13/3$</td><td>$1/3$</td></tr></table>	5	$13/3$	$1/3$			
5	$13/3$	$1/3$					
4	e' <table border="1"><tr><td>5</td><td>3</td><td>2</td></tr></table>	5	3	2			
5	3	2					
5	\emptyset						

- the edge c is erased, because it is parallel with the direction of scanning,
- edges d and e are shortened to edges d' and e' , respectively, because they contain a non-extremal vertex and lie in the upper halfplane with respect to the line passing through the vertex



Processing

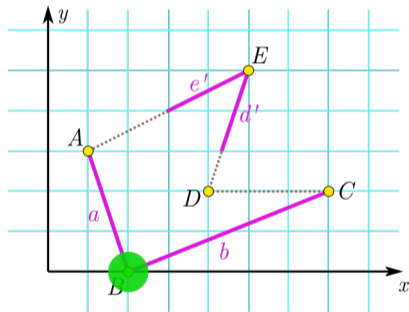
$$y = y_{min} = 0$$

- 1 $TAE := TAE \cup \{a, b\} = \{a, b\}$
 $TE := TE \setminus \{a, b\} = \{d', e'\}$
- 2 $\text{Sort}_x(TAE) = [a, b]$
- 3 $\text{Draw}([a, b], 0) = \{(\text{Round}(2), 0)^\top\}$
- 4 $TAE := TAE \setminus \{\text{edges} \mid y_{max} = y\} = \{a, b\} \setminus \emptyset = \{a, b\}$
- 5 a

3	5/3	-1/3
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 b

2	9/2	5/2
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- 6 $y := y + 1 = 1$



Processing

$$y = 1$$

$$1 \quad TAE := TAE \cup \emptyset = \{a, b\}$$

$$TE := TE \setminus \emptyset = \{d', e'\}$$

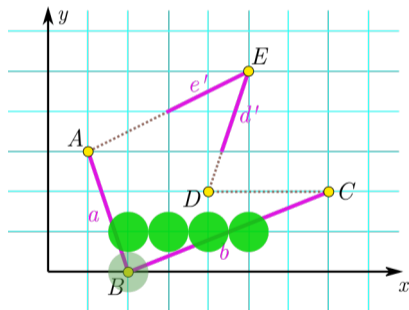
$$2 \quad \text{Sort}_x(TAE) = [a, b]$$

$$3 \quad \text{Draw}([a, b], 1) = \\ \{(\text{Round}(\frac{5}{3}), 1)^\top, \dots, (\text{Round}(\frac{9}{2}), 1)^\top\} = \\ \{(2, 1)^\top, (3, 1)^\top, (4, 1)^\top, (5, 1)^\top\}$$

$$4 \quad TAE := \{a, b\} \setminus \emptyset = \{a, b\}$$

$$5 \quad a \begin{bmatrix} 3 & 4/3 & -1/3 \end{bmatrix} \quad b \begin{bmatrix} 2 & 7 & 5/2 \end{bmatrix}$$

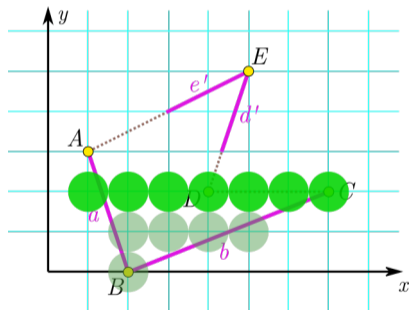
$$6 \quad y := y + 1 = 2$$



Processing

$$y = 2$$

- 1 $TAE := TAE \cup \emptyset = \{a, b\}$
 $TE := TE \setminus \emptyset = \{d', e'\}$
- 2 $\text{Sort}_x(TAE) = [a, b]$
- 3 $\text{Draw}([a, b], 2) =$
 $\{(\text{Round}(4/3), 2)^\top, \dots, (\text{Round}(7), 2)^\top\} =$
 $\{(1, 2)^\top, \dots, (7, 2)^\top\}$
- 4 $TAE := \{a, b\} \setminus \{b\} = \{a\}$
- 5 $a \begin{bmatrix} 3 & 1 & -1/3 \end{bmatrix}$
- 6 $y := y + 1 = 3$

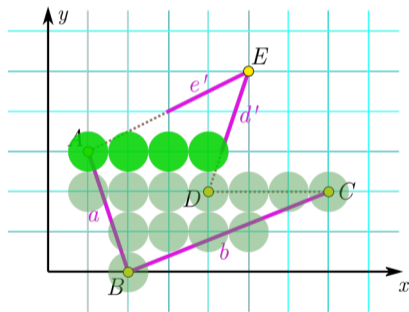


Processing

$$y = 3$$

- 1 $TAE := TAE \cup \{d'\} = \{a, d'\}$
 $TE := TE \setminus \{d'\} = \{e'\}$
- 2 $\text{Sort}_x(TAE) = [a, d']$
- 3 $\text{Draw}([a, d'], 3) =$
 $\{(\text{Round}(1), 3)^\top, \dots, (\text{Round}(13/3), 3)^\top\} =$
 $\{(1, 3)^\top, \dots, (4, 3)^\top\}$
- 4 $TAE := \{a, d'\} \setminus \{a\} = \{d'\}$
- 5 d'

5	$14/3$	$1/3$
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- 6 $y := y + 1 = 4$



Processing

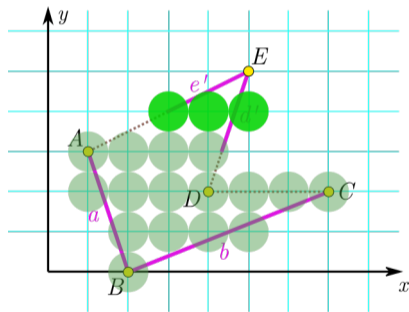
$$y = 4$$

- 1 $TAE := TAE \cup \{e'\} = \{d', e'\}$
 $TE := TE \setminus \{e'\} = \emptyset$
- 2 $\text{Sort}_x(TAE) = [e', d']$
- 3 $\text{Draw}([e', d'], 4) =$
 $\{(\text{Round}(3), 4)^\top, \dots, (\text{Round}(14/3), 4)^\top\} =$
 $\{(3, 4)^\top, \dots, (5, 4)^\top\}$
- 4 $TAE := \{d', e'\} \setminus \emptyset = \{d', e'\}$
- 5 d'

5	5	$1/3$
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 e'

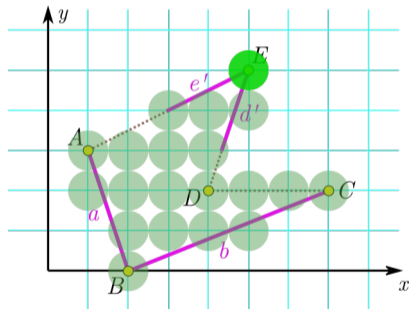
5	5	2
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- 6 $y := y + 1 = 5$



Processing

$$y = 5$$

- 1 $TAE := TAE \cup \emptyset = \{d', e'\}$
 $TE = \emptyset$
- 2 $\text{Sort}_x(TAE) = [e', d']$
- 3 $\text{Draw}([e', d'], 5) = \{(5, 5)^\top\}$
- 4 $TAE := \{d', e'\} \setminus \{d', e'\} = \emptyset$
- 5 CONTINUE
- 6 $y := y + 1 = 6$



Processing

$$y = 6$$

$$1 \quad TAE = \emptyset$$

$$TE = \emptyset$$

\hookrightarrow **END**

