Clipping

Cohen-Sutherland and Cyrus-Beck algorithms

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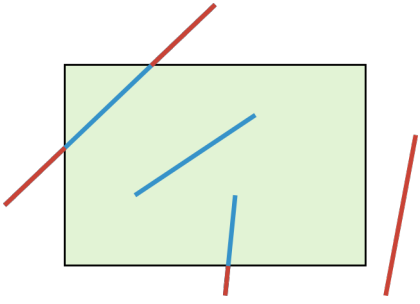
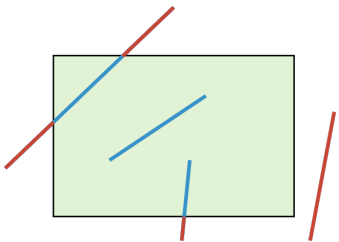
Motivation

- The aim is to remove each (part of) object which is outside the viewing area.
- Only the line segments need to be clipped.
  - The boundary of a polygon is also created by line segments.
- Usually we need to process the large number of line segments $\longrightarrow$ small number of operations
  - How to find the intersection of a line and the window efficiently?

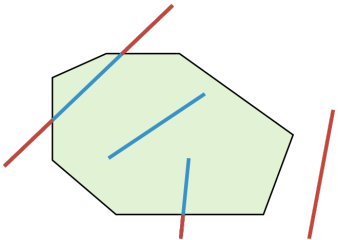


Algorithms

The algorithms may be classified with respect to the shape of the viewing area (clip window):



RECTANGLE
Cohen-Sutherland, Liang-Barsky, Nichol-Lee-Nichol



CONVEX POLYGON
Cyrus-Beck

Cohen-Sutherland

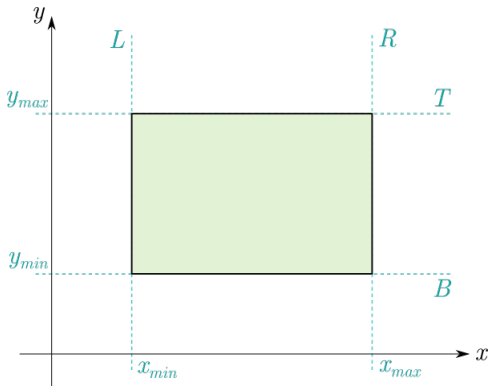
- Works only for the rectangular (axis-aligned) clip window

INPUT:
- clip window (given by values $x_{min}, x_{max}, y_{min}, y_{max}$,
- list of line segments (given by endpoints).

PROCESSING: For each line segment CD:
1. encode the points C and D,
2. decide if the line segment is **accepted / rejected / clipped**.

OUTPUT:
- list of the clipped line segments (their endpoints), drawing.

Cohen-Sutherland – INPUT – clip window



- Lines $x = x_{min}, x = x_{max}, y = y_{min}$ and $y = y_{max}$ divide the plane into 9 regions.
- Let us introduce the notation:

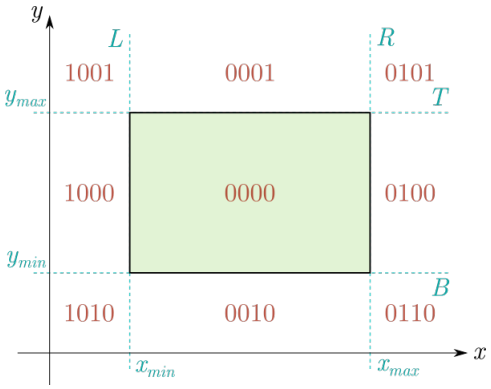
  if $x < x_{min}$, then $L = 1$; else $L = 0$

  if $x > x_{max}$, then $R = 1$; else $R = 0$

  if $y < y_{min}$, then $B = 1$; else $B = 0$

  if $y > y_{max}$, then $T = 1$; else $T = 0$

Cohen-Sutherland – INPUT – clip window



- Using this notation we can encode each of the region using only 4 bits.
  - We need to choose the order of bits, e.g. $LRBT$.
- Each region has unique encoding, i.e. to determine the position of a point with respect to the regions, we need to determine its $LRBT$ code.

Cohen-Sutherland – PROCESSING

For each line segment CD:

1. get the $LRBT$ code for points C and D, i.e. code(C), code(D),
2. now, three situations may occur

Trivial accept – $\big(\text{code}(C)\ ||\ \text{code}(D)\big) = 0000$

both endpoints lie inside the region 0000,
$\hookrightarrow$ draw the line segment AB

Trivial reject – $\big(\text{code}(C)\ \&\ \text{code}(D)\big) \neq 0000$

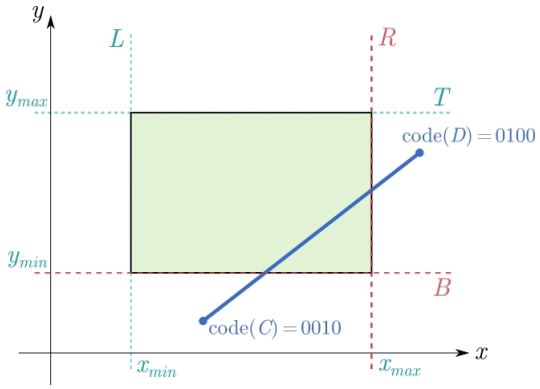
both endpoints lie outside the region 0000,
$\hookrightarrow$ drop the line segment AB

Test clipping – $\big(\text{code}(C)\ \&\ \text{code}(D)\big) = 0000$

the line segments intersects the region 0000,
$\hookrightarrow$ perform clipping

Cohen-Sutherland – PROCESSING – test clipping

The line segment CD is clipped by those lines, which have the bit value 1 (e.g. if one of the points C or D has L-bit equal to 1, we clip it by the respective line $x = x_{min}$).



Cohen-Sutherland – PROCESSING – test clipping

The coordinates of the endpoints of the clipped line:

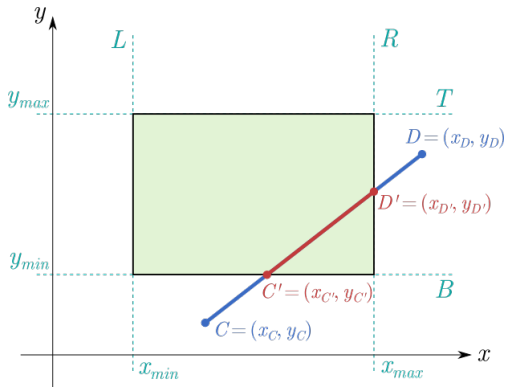
- Clipping by R:

$$x_{D'} = x_{max}$$
$$y_{D'} = y_D + \frac{y_C - y_D}{x_C - x_D}(x_{max} - x_D)$$

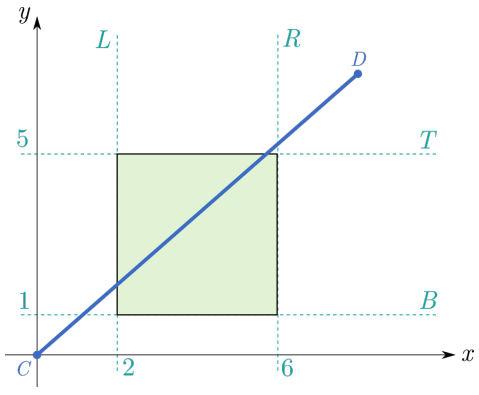
- Clipping by B:

$$x_{C'} = x_C + \frac{x_C - x_D}{y_C - y_D}(y_{min} - y_C)$$
$$y_{C'} = y_{min}$$

Clipping by L and T may be derived analogically.



Consider the line segment CD, whose endpoints have coordinates $C = (0, 0)^\top, D = (8, 7)^\top$. Clip CD by the axis-aligned window given by its corners $(2, 1)^\top$ and $(6, 5)^\top$ in the clipping order $LRBT$.

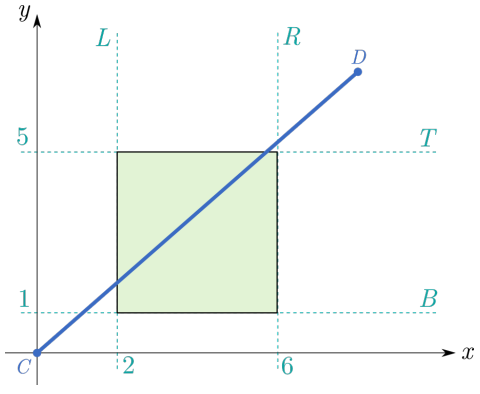


To encode the endpoints of the segment CD we firstly determine the rules for the clipping lines, i. e.
if $x < 2$, then $L = 1$; else $L = 0$
if $x > 6$, then $R = 1$; else $R = 0$
if $y < 1$, then $B = 1$; else $B = 0$
if $y > 5$, then $T = 1$; else $T = 0$.

The codes of the endpoints are:

$$\text{code}(C) = 1010 \quad \text{code}(D) = 0101.$$

Since $\text{code}(C)\ \&\ \text{code}(D) = 0000$, the segment can not be neither accepted nor rejected. Thus it needs to be clipped.



Before we start clipping, we initialize endpoints of the clipped segments to the endpoints of the original segments, i. e.

$$C' \leftarrow C = (2,1)^\top, \quad D' \leftarrow D = (8,7)^\top$$

$$\text{code}(C') = 1010 \quad \text{code}(D') = 0101.$$

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clip L:

$\text{code}_L(C') = 1 \rightsquigarrow$ CLIP

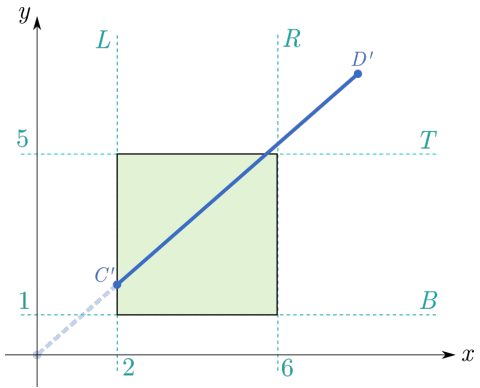
$x' = 2$

$y' = 0 + \frac{7-0}{8-0}(2-0) = \frac{7}{4}$

$C' \leftarrow \left(2, \frac{7}{4}\right)$

$\text{code}(C') \leftarrow 0000$

$\text{code}_L(D') = 0 \rightsquigarrow$ NO CLIP



clip R:

$\text{code}_R(C') = 0 \rightsquigarrow$ NO CLIP

$\text{code}_R(D') = 1 \rightsquigarrow$ CLIP

$x' = 6$

$y' = 7 + \dfrac{7-0}{8-0}(6-8) = \dfrac{21}{4}$

$D' \leftarrow \left(6, \dfrac{21}{4}\right)$

$\text{code}(D') \leftarrow 0001$

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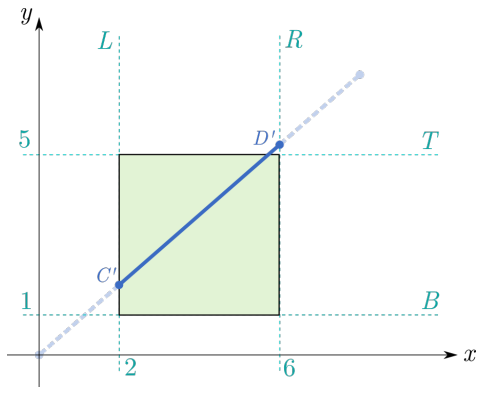
clip B:

$\text{code}_B(C') = 0 \rightsquigarrow$ NO CLIP

$\text{code}_B(D') = 0 \rightsquigarrow$ NO CLIP



clip T:

code$_T(C') = 0 \rightsquigarrow$ NO CLIP

code$_T(D') = 1 \rightsquigarrow$ CLIP

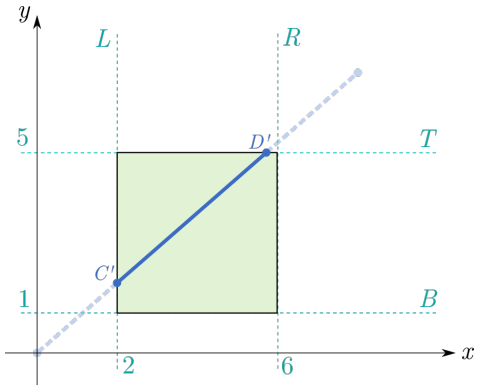
$x' = 6 + \frac{8-0}{7-0}(5 - \frac{21}{4}) = \frac{40}{7}$

$y' = 5$

$D' \leftarrow \left(\frac{40}{7}, 5\right)$

code$(D') \leftarrow 0000$

We finished clipping and see, that both endpoints have codes 0000, i. e. they lie inside the clipping window.



Cyrus-Beck

- Works for any convex polygon (i.e. including rectangles)

INPUT:
- an oriented convex polygon given by its edges $\mathbf{e}_i$ and normal vectors,
- list of oriented line segments (given by endpoints).

PROCESSING: For each oriented line segment AB:

1. for each edge $\mathbf{e}_i$, clip AB by the edge $\mathbf{e}_i$.

OUTPUT:
- list of the clipped line segments (their endpoints), drawing.

Cyrus-Beck – input line segment

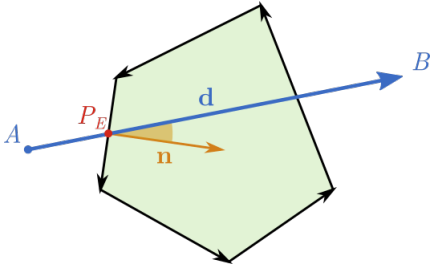
- The segment AB is expressed by its parametric equation

$$P(t) = A + \mathbf{d}t, \qquad \mathbf{d} = B - A, \qquad t \in \langle 0, 1 \rangle.$$

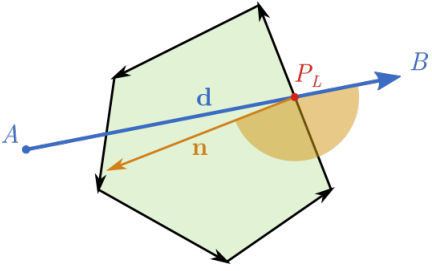
- We are searching for:
  - $\max\{t_E\} \in \langle 0, 1 \rangle$ – the maximal t value, where the line segments **enters** the clip window, i.e. the entering point P_E,
  - $\min\{t_L\} \in \langle 0, 1 \rangle$ – the minimal t value, where the line segments **leaves** the clip window, i.e. the leaving point P_L,

Cyrus-Beck – entering and leaving points

The entering and leaving points may be classified as follows:



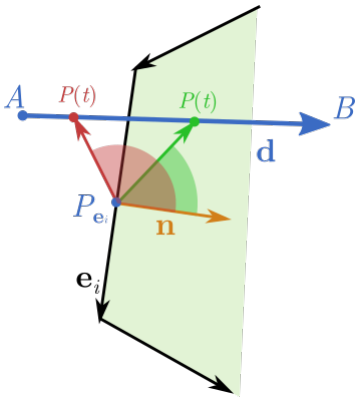
point P_E is entering if $\langle \mathbf{n}, \mathbf{d} \rangle \geq 0$



point P_L is leaving if $\langle \mathbf{n}, \mathbf{d} \rangle \leq 0$

Cyrus-Beck – position of a point with respect to the edge

The entering and leaving points may be classified as follows:



The point $P(t) \in AB$ is:

- exterior, if $\sphericalangle(P(t) - P_{\mathbf{e}_i}, \mathbf{n}) > {}^{\pi}/_{2}$, i.e. $\langle P(t) - P_{\mathbf{e}_i}, \mathbf{n} \rangle < 0$
- interior, if $\sphericalangle(P(t) - P_{\mathbf{e}_i}, \mathbf{n}) < {}^{\pi}/_{2}$, i.e. $\langle P(t) - P_{\mathbf{e}_i}, \mathbf{n} \rangle > 0$

Cyrus-Beck – a point lying on the edge

The point $P(t)$ lies on the edge $\mathbf{e}_i$, if $\sphericalangle(P(t) - P_{\mathbf{e}_i}, \mathbf{n}) = {}^{\pi}/_{2}$, i.e. $\langle P(t) - P_{\mathbf{e}_i}, \mathbf{n} \rangle = 0$. By expanding the expression we can express the parameter t and differentiate between three situations:

- $\langle \mathbf{d}, \mathbf{n} \rangle \neq 0$ and $t = \dfrac{\langle P_{\mathbf{e}_i} - A, \mathbf{n} \rangle}{\langle \mathbf{d}, \mathbf{n} \rangle} \rightsquigarrow AB$ and $\mathbf{e}_i$ are intersecting $\rightsquigarrow$ CLIP,
- $\langle \mathbf{d}, \mathbf{n} \rangle = 0$ and $\langle P_{\mathbf{e}_i} - A, \mathbf{n} \rangle = 0 \rightsquigarrow \mathbf{e}_i$ belongs to the **line** $AB \rightsquigarrow$ NO CLIP,
- $\langle \mathbf{d}, \mathbf{n} \rangle = 0$ and $\langle P_{\mathbf{e}_i} - A, \mathbf{n} \rangle \neq 0 \rightsquigarrow \mathbf{e}_i$ is parallel with $AB \rightsquigarrow$ NO CLIP,

Cyrus-Beck – clipping

Knowing, that $P(t) \in \mathbf{e}_i \Leftrightarrow t = \frac{\langle P_{\mathbf{e}_i} - A, \mathbf{n} \rangle}{\langle \mathbf{d}, \mathbf{n} \rangle}$, we say:

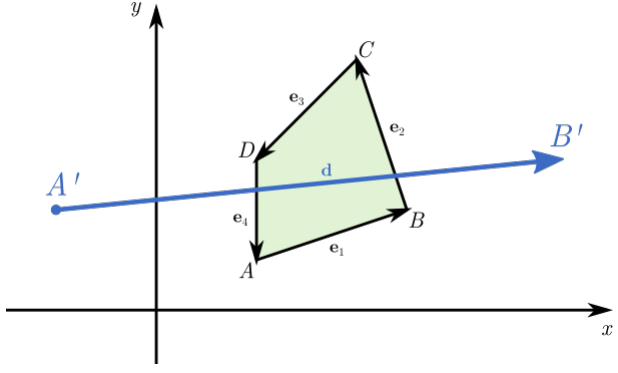
- if $\langle \mathbf{d}, \mathbf{n} \rangle > 0$, t is the *entering parameter candidate*, denoted by t_E, and the point $P(t_E)$ is the *entering point candidate*,
- if $\langle \mathbf{d}, \mathbf{n} \rangle < 0$, t is the *leaving parameter candidate*, denoted by t_L, and the point $P(t_L)$ is the *leaving point candidate*,

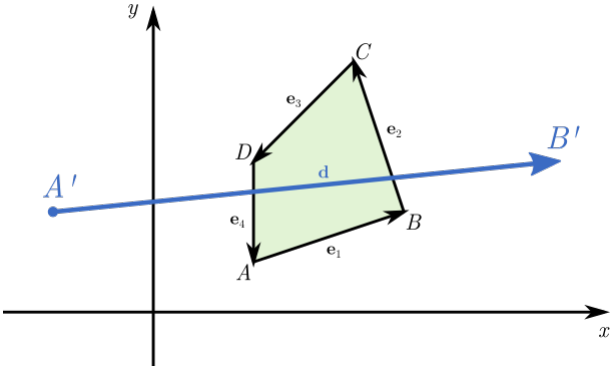
After we finished the clipping, we need to check if $t_E < t_L$, so the clipped line actually lies in the clip window

Clip the line segment $A'B'$ into the clip window, given by vertices

$$A = (2,1)^\top,\ B = (5,2)^\top,\ C = (4,5)^\top,\ D = (2,3)^\top,$$

where the endpoints of the segment are $A' = (-2,2)^\top$ and $B' = (8,3)^\top$. The clipping is performed using the Cyrus-Beck algorithm.





PREPROCESSING:

1. choose the orientation of the clip polygon – **positive** (CCW) – inward-pointing normals
2. set the direction vector of $A'B'$ $\mathbf{d} := B' - A' = (10, 1)^\top$
3. initialize the entering and leaving parameter $t_E = 0,\ t_L = 1$.

CLIP $\mathbf{e}_1$:

1. Choose the normal $\mathbf{n}_1$ for $\mathbf{e}_1$:

   $$\mathbf{n}_1 \sim B - A = (3, 1)$$
   $$\mathbf{n}_1 = (-1, 3)$$

   Choose the (arbitrary) point P_1 belonging to $\mathbf{e}_1$:

   $$P_1 = \frac{A + B}{2} = ({}^7/_2, {}^3/_2)$$ (we choose the midpoint)

2. Classify the type of parameter t:

   $$\langle \mathbf{d}, \mathbf{n}_1 \rangle = \langle (10, 1), (-1, 3) \rangle = -7 < 0$$
   t is the leaving candidate ($t \rightsquigarrow t_L$)

3. Compute the value of t:

   $$t = \frac{\langle P_1 - A', \mathbf{n}_1 \rangle}{\langle \mathbf{d}, \mathbf{n}_1 \rangle} = \frac{-\frac{14}{2}}{-7} = 1.$$

4. Refresh the t_L value:

   $$t_L = \min\{t_L, t\} = \min\{1, 1\} = 1$$

CLIP $\mathbf{e}_2$:

1. $\mathbf{n}_2 \sim C - B = (-1, 3)$
   $\mathbf{n}_2 = (-3, -1)$
   $P_2 = \dfrac{B + C}{2} = ({}^{9}/_{2}, {}^{7}/_{2})$
2. $\langle \mathbf{d}, \mathbf{n}_2 \rangle = \langle (10, 1), (-3, -1) \rangle = -31 < 0$
   $t \rightsquigarrow t_L$
3. $t = \dfrac{\langle P_2 - A', \mathbf{n}_2 \rangle}{\langle \mathbf{d}, \mathbf{n}_2 \rangle} = \dfrac{-\frac{42}{2}}{-31} = \dfrac{21}{31}.$
4. $t_L = \min\{t_L, t\} = \min\left\{1, \dfrac{21}{31}\right\} = \dfrac{21}{31}$

CLIP $\mathbf{e}_3$:

1. $\mathbf{n}_3 \sim D - C = (-2, -2)$
   $\mathbf{n}_3 = (2, -2)$
   $P_3 = \dfrac{C + D}{2} = (3, 4)$
2. $\langle \mathbf{d}, \mathbf{n}_3 \rangle = \langle (10, 1), (2, -2) \rangle = 18 > 0$
   $t \rightsquigarrow t_E$
3. $t = \dfrac{\langle P_3 - A', \mathbf{n}_3 \rangle}{\langle \mathbf{d}, \mathbf{n}_3 \rangle} = \dfrac{6}{18} = \dfrac{1}{3}.$
4. $t_E = \max\{t_E, t\} = \max\left\{0, \dfrac{1}{3}\right\} = \dfrac{1}{3}$

CLIP $\mathbf{e}_4$:

1. $\mathbf{n}_4 \sim A - D = (0, -2)$
   $\mathbf{n}_4 = (2, 0)$
   $P_4 = \dfrac{D + A}{2} = (2, 2)$
2. $\langle \mathbf{d}, \mathbf{n}_4 \rangle = \langle (10, 1), (2, 0) \rangle = 20 > 0$
   $t \rightsquigarrow t_E$
3. $t = \dfrac{\langle P_4 - A', \mathbf{n}_4 \rangle}{\langle \mathbf{d}, \mathbf{n}_4 \rangle} = \dfrac{8}{20} = \dfrac{2}{5}.$
4. $t_E = \max\{t_E, t\} = \max\left\{\dfrac{1}{3}, \dfrac{2}{5}\right\} = \dfrac{2}{5}$

After we finnished clipping by the edges, we need to check if

$$\langle 0, 1 \rangle \ni \frac{2}{5} = t_E < t_L = \frac{21}{31} \in \langle 0, 1 \rangle,$$

which is obviously true. Now we can finally compute the endpoints P_E, P_L of the clipped line segment by inserting it to the equation of the line segment $A'B'$:

$$P(t) = A' + \mathbf{d}t = (-2, 2) + (10, 1)t,$$

$$P_E = P(t_E) = (2, {}^{12}/_{5}),$$

$$P_L = P(t_L) = ({}^{148}/_{31}, {}^{83}/_{31}).$$