This paper contains the list of problems from the course ”Computer graphics (1)”. Some of these problems were solved on the lessons, some occurred in the homework, midterm or the final exam. The problems should serve for your preparation for the written exams. The problems are either in slovak or english language.

1 Review of basic concepts

1. Tell the elements of the matrix $\mathbf{U} \cdot \mathbf{V}$, if

$$\mathbf{U} = \begin{pmatrix} 1 & 7 & 0 \\ 5 & -3 & 6 \end{pmatrix} \quad \text{and} \quad \mathbf{V} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}.$$

What is the information about its dimensions?

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2. Consider the matrix

$$\mathbf{U} = \begin{pmatrix} 3 & 1 & 0 \\ -1 & 2 & 1 \\ 0 & -1 & 1 \end{pmatrix}.$$

- Find the determinant $\det(\mathbf{U})$ of the matrix
- Find the inverse $\mathbf{U}^{-1}$ using elementary row operations, if possible
- Find the inverse $\mathbf{U}^{-1}$ using the adjugate of a matrix, if possible

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3. For what values $(x, y) \in \mathbb{R}^2$ is the matrix

$$\mathbf{U} = \begin{pmatrix} 1 & 1 & -1 \\ 1 & x & 2 \\ -1 & 0 & y \end{pmatrix}$$

singular? Describe the set of such values in detail and also, depict it graphically.

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4. Let $a, b \in \mathbb{R}$,

$$\mathbf{U} = \begin{pmatrix} 1 & 0 & -1 \\ 0 & a & 0 \\ 2 & 1 & b \end{pmatrix} \quad \text{and} \quad \mathbf{U}^{-1} = \begin{pmatrix} * & * & * \\ * & 3 & * \\ * & -\frac{6}{5} & * \end{pmatrix}.$$

Determine the values a, b. Use the adjugate of a matrix in your computations.

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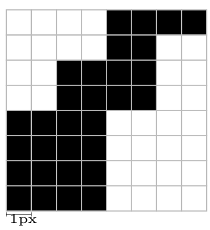
2 Space partitioning and meshes

1. Record the following monochromatic image using quadtree in both tree and line notation:



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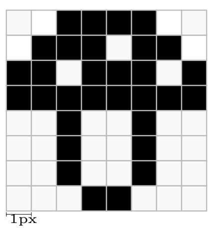
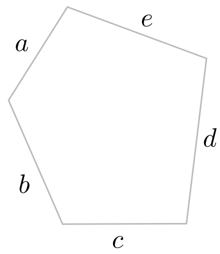
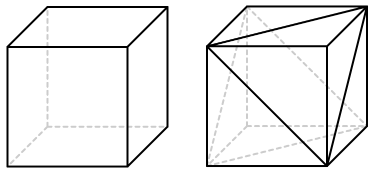
2. Construct the quadtree for the following monochromatic image in the tree notation:







3. Create a BSP tree for a pentagon. Use the edge a for creating the first subdividing hyperplane. Is the created tree balanced? Is there a way to reduce the depth of the tree?

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4. Determine the genus g of the cube represented as a quadrilateral mesh and also as a triangle mesh:

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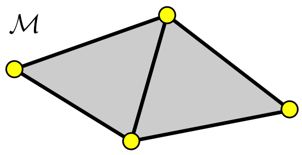
5. Construct the table of (half)edges for the mesh $\mathcal{M}$, represented as

   - winged-edge data structure,
   - halfedge data structure.

   Also, choose the notation for vertices, edges and faces.



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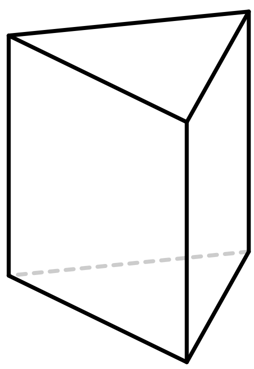
6. Create the table of halfedges for the prism depicted below, represented as a halfedge data structure. Choose the notation for vertices, edges and faces:



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3 Affine transformations and projections

1. Consider a point

$$X = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

and a line p defined by a point P and a vector $\vec{u}$ with coordinates

$$P = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}, \quad \vec{u} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}.$$

What are the coordinates of a point $\hat{X}$, which is the result of the rotation of the point X about the line p by the angle $\varphi = \frac{2}{3}\pi$? Perform the computation using

- quaternions,
- affine transformations.

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2. We are working in the Cartesian coordinate system $\langle O, \vec{e_0}, \vec{e_1}\rangle$, where a point X has coordinates

$$X = \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$

Tell the coordinates of the point X in a coordinate system $\langle A, B - A, C - A\rangle$, where

$$A = \begin{pmatrix} -1 \\ 0 \end{pmatrix}, \ B = \begin{pmatrix} 0 \\ 2 \end{pmatrix}, \ C = \begin{pmatrix} -2 \\ 1 \end{pmatrix}.$$

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3. What is the result of the rotation of the point

$$X = \begin{pmatrix} 4 \\ 0 \\ -2 \end{pmatrix}$$

about the line p, which passes through the points

$$P = \begin{pmatrix} 3 \\ \sqrt{2} \\ -1 \end{pmatrix}, \ Q = \begin{pmatrix} 2 \\ 2\sqrt{2} \\ 0 \end{pmatrix},$$

by the angle $\varphi = \frac{\pi}{3}$? The rotation is computed using quaternions.

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4. Bod $\hat{X}$, ktorý sme získali otočením bodu X okolo priamky p o uhol φ, je daný transformáciou:

$$\hat{X} = \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos(\pi/3) & -\sin(\pi/3) & 0 & 0 \\ \sin(\pi/3) & \cos(\pi/3) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos(\pi/6) & 0 & \sin(\pi/6) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\pi/6) & 0 & \cos(\pi/6) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos(\pi/4) & -\sin(\pi/4) & 0 & 0 \\ \sin(\pi/4) & \cos(\pi/4) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
$$\begin{pmatrix} \cos(-\pi/6) & 0 & \sin(-\pi/6) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(-\pi/6) & 0 & \cos(-\pi/6) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos(-\pi/3) & -\sin(-\pi/3) & 0 & 0 \\ \sin(-\pi/3) & \cos(-\pi/3) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ -2 \\ 1 \\ 1 \end{pmatrix}.$$

(a) určite súradnice bodu X,

(b) určite veľkosť uhla φ,

(c) určite súradnice bodu P a **jednotkového** smerového vektora $\mathbf{u}$, ktorý definuje priamku $p := \langle P, \mathbf{u}\rangle$.

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5. Nech afinná transformácia $\mathbb{M}$ zobrazí krivku $\mathcal{K}_1$ na krivku $\mathcal{K}_2$, ktoré sú zadané nasledovne:

$$\mathcal{K}_1 := \left\{(x, y) \in \mathbb{R}^2 \;\middle|\; \frac{(x-1)^2}{4} + \frac{(y+1)^2}{1} = 1\right\},$$

$$\mathcal{K}_2 : \begin{aligned} x(t) &= 1 + 4\cos t - 2\sin t \\ y(t) &= 1 - 2\cos t - \sin t, \qquad t \in \langle 0, 2\pi). \end{aligned}$$

Zapíšte $\mathbb{M}$ ako zloženie **otočenia**, **škálovania** a **posunutia** (v tomto poradí).

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6. Calculate the coefficients of the matrix $\mathbf{M}$ corresponding to a transformation that maps the square $ABCD$ into the rectangle $A'B'C'D'$ if

$$A = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, B = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, C = \begin{pmatrix} 0 \\ 3 \end{pmatrix}, D = \begin{pmatrix} -1 \\ 2 \end{pmatrix}, \qquad A' = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, B' = \begin{pmatrix} 0 \\ 4 \end{pmatrix}, C' = \begin{pmatrix} -2 \\ 4 \end{pmatrix}, D' = \begin{pmatrix} -2 \\ 0 \end{pmatrix}.$$

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7. Nech transformácia $\mathbb{M}$ zobrazí dané body nasledovne:

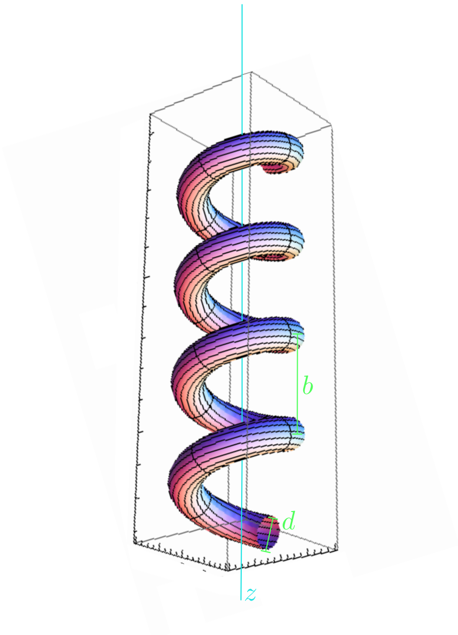
$$\begin{pmatrix}0\\0\end{pmatrix} \mapsto \begin{pmatrix}1\\-1\end{pmatrix}, \quad \begin{pmatrix}1\\0\end{pmatrix} \mapsto \begin{pmatrix}1\\2\end{pmatrix}, \quad \begin{pmatrix}0\\1\end{pmatrix} \mapsto \begin{pmatrix}3\\-1\end{pmatrix}.$$

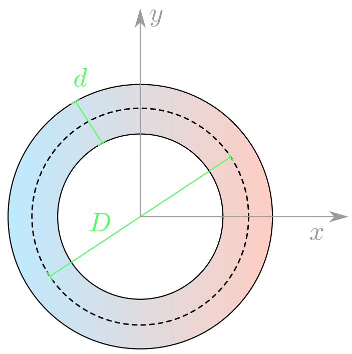
   Zapíšte $\mathbb{M}$ ako zloženie:

   (a) škálovania $\to$ rotácie $\to$ posunutia,

   (b) posunutia $\to$ škálovania $\to$ rotácie.

8. Find the parametrization of the spring depicted in the figure below, using the sweeping method. The created surface might be characterized as an infinite tubular wire with circular section of width d, wrapped around a cylinder with the axis identical with the coordinate axis z and diameter D. The pitch of the spring, i.e. height of complete helix turn, is b.





Top view

9. *(2 pts)* Consider a plane π determined by points

$$A = \begin{pmatrix}1\\0\\0\end{pmatrix}, \quad B = \begin{pmatrix}0\\2\\0\end{pmatrix}, \quad C = \begin{pmatrix}0\\0\\1\end{pmatrix},$$

   and a triangle XYZ with its vertices being

$$X = \begin{pmatrix}1\\1\\2\end{pmatrix}, \quad Y = \begin{pmatrix}1\\2\\1\end{pmatrix}, \quad Z = \begin{pmatrix}1\\2\\2\end{pmatrix}.$$

   Choose a coordinate system in the plane π and determine the coordinates of vertices of a triangle $X'Y'Z'$, which is a result of central projection of the triangle XYZ onto the plane π with the centre of projection being the origin $O = (0,0,0)^\top$.

4 Filling

1. Fill the polygon $ABCDE$ using the scanline algorithm, where

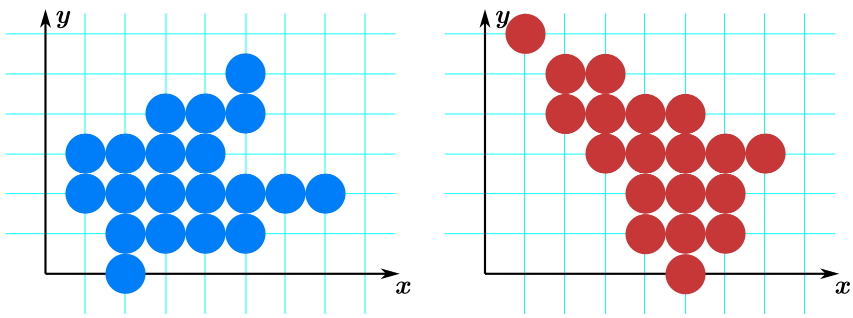
$$A = \begin{pmatrix}1\\3\end{pmatrix}, \quad B = \begin{pmatrix}2\\0\end{pmatrix}, \quad C = \begin{pmatrix}7\\2\end{pmatrix}, \quad D = \begin{pmatrix}4\\2\end{pmatrix}, \quad E = \begin{pmatrix}5\\5\end{pmatrix}.$$

2. Rasterize the quadrilateral $ABCD$ using the scanline algorithm with the scanning line moving in the y-direction, if
$$A = \begin{pmatrix}3\\3\end{pmatrix}, B = \begin{pmatrix}1\\1\end{pmatrix}, C = \begin{pmatrix}3\\0\end{pmatrix}, D = \begin{pmatrix}5\\1\end{pmatrix}.$$

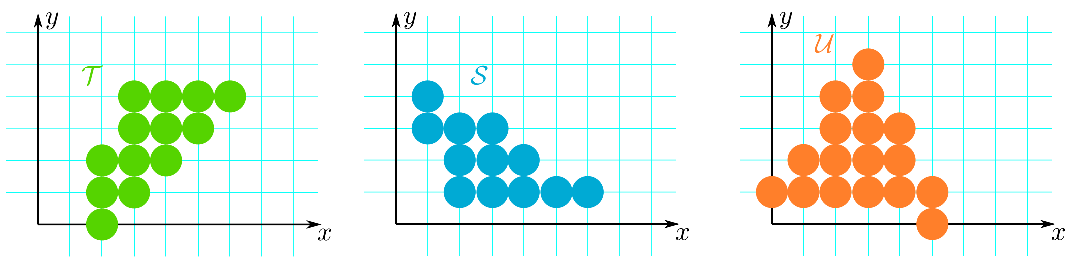
3. Fill the polygon $ABCDE$ using the scanline algorithm (tell the coordinates of all pixels, that are drawn on a display), where
$$A = \begin{pmatrix}8\\4\end{pmatrix}, \quad B = \begin{pmatrix}8\\1\end{pmatrix}, \quad C = \begin{pmatrix}11\\1\end{pmatrix}, \quad D = \begin{pmatrix}2\\5\end{pmatrix}, \quad E = \begin{pmatrix}4\\1\end{pmatrix}.$$

5 Visibility

1. Draw polygons $\mathcal{Q}, \mathcal{T}$ using the Z-buffer algorithm if
$$\mathcal{Q}:\ A = \begin{pmatrix}1\\3\\-3\end{pmatrix}, B = \begin{pmatrix}2\\0\\-1\end{pmatrix}, C = \begin{pmatrix}7\\2\\-8\end{pmatrix}, D = \begin{pmatrix}5\\5\\-9\end{pmatrix},$$
$$\mathcal{T}:\ F = \begin{pmatrix}1\\6\\-8\end{pmatrix}, G = \begin{pmatrix}5\\0\\-2\end{pmatrix}, H = \begin{pmatrix}7\\3\\-8\end{pmatrix},$$
and rasterized form of these polygons are:



2. Draw triangles $\mathcal{T}, \mathcal{S}, \mathcal{U}$ defined by vertices ABC, DEF, and GHI, respectively, using the Z-buffer technique, if
$$\mathcal{T}:\ A = \begin{pmatrix}2\\0\\-3\end{pmatrix}, \quad B = \begin{pmatrix}6\\4\\-3\end{pmatrix}, \quad C = \begin{pmatrix}3\\4\\0\end{pmatrix},$$
$$\mathcal{S}:\ D = \begin{pmatrix}2\\1\\-1\end{pmatrix}, \quad E = \begin{pmatrix}6\\1\\-5\end{pmatrix}, \quad F = \begin{pmatrix}1\\4\\-3\end{pmatrix},$$
$$\mathcal{U}:\ G = \begin{pmatrix}0\\1\\0\end{pmatrix}, \quad H = \begin{pmatrix}5\\0\\-4\end{pmatrix}, \quad I = \begin{pmatrix}3\\5\\-7\end{pmatrix}.$$
The rasterized triangles are depicted below:



3. Determine visibility of faces and edges of the tetrahedron $ABCD$ using the Roberts' algorithm, if

$$A = \begin{pmatrix}0\\0\\0\end{pmatrix}, B = \begin{pmatrix}1\\1\\0\end{pmatrix}, C = \begin{pmatrix}0\\2\\0\end{pmatrix}, D = \begin{pmatrix}0\\1\\1\end{pmatrix},$$

and the viewer stands in the point $R = (5,4,3)^\top$ and looks at the origin of the coordinate system O.

4. Using the back face culling, decide, which <u>faces</u> of the box $\mathcal{B}$ are visible from the point $W = (2,0,2)^\top$, if we are looking at the centre of the box $\mathcal{B}$, given by vertices

$$A = \begin{pmatrix}0\\0\\0\end{pmatrix}, B = \begin{pmatrix}1\\0\\0\end{pmatrix}, C = \begin{pmatrix}1\\2\\0\end{pmatrix}, D = \begin{pmatrix}0\\2\\0\end{pmatrix}, E = \begin{pmatrix}0\\0\\1\end{pmatrix}, F = \begin{pmatrix}1\\0\\1\end{pmatrix}, G = \begin{pmatrix}1\\2\\1\end{pmatrix}, H = \begin{pmatrix}0\\2\\1\end{pmatrix}.$$

5. Consider a rectangular pyramid $ABCDV$ given by vertices

$$A = \begin{pmatrix}-2\\1\\0\end{pmatrix}, \quad B = \begin{pmatrix}2\\1\\0\end{pmatrix}, \quad C = \begin{pmatrix}2\\-1\\0\end{pmatrix}, \quad D = \begin{pmatrix}-2\\-1\\0\end{pmatrix}, \quad V = \begin{pmatrix}0\\0\\2\end{pmatrix}.$$

Using the Robert's algorithm for back face culling, choose which edges are potentially visible and invisible, if the viewer stands in the point $P = (-3,3,1)^\top$ and looks towards the origin of the positively oriented coordinate system.

6 Clipping

1. Clip the line AB, $A = (0,0)^\top, B = (8,7)^\top$ into the axial window defined by its two corners $(2,1)^\top$ and $(6,5)^\top$, using the Cohen-Sutherland algorithm with the binary code $LRDU$.

2. Clip the line segments AB, CD and EF using the Cohen-Sutherland algorithm (with order of the borders $TBRL$ - top, bottom, right, left) into the axial window $\mathcal{W}$ defined by its corners $(2,2)^\top, (5,4)^\top$ if

$$A = \begin{pmatrix}3\\1\end{pmatrix}, B = \begin{pmatrix}7\\5\end{pmatrix}, C = \begin{pmatrix}3\\3\end{pmatrix}, D = \begin{pmatrix}4\\3\end{pmatrix}, E = \begin{pmatrix}1\\5\end{pmatrix}, F = \begin{pmatrix}1\\3\end{pmatrix}.$$

3. Clip the lines AB, CD and EF to the window determined by points $P_1 = (1,1)^\top, P_2 = (11,6)^\top$, using the Cohen-Sutherland algorithm with <u>$LDRU$</u> binary coding, if

$$A = \begin{pmatrix}4\\0\end{pmatrix}, B = \begin{pmatrix}12\\8\end{pmatrix}, C = \begin{pmatrix}-1\\4\end{pmatrix}, D = \begin{pmatrix}5\\4\end{pmatrix}, E = \begin{pmatrix}-3\\2\end{pmatrix}, F = \begin{pmatrix}5\\-2\end{pmatrix}.$$

4. Clip the line CD into the polygonal window $PRST$ using the algorithm *Cyrus-Beck*, if

$$C = \begin{pmatrix}-1\\1\end{pmatrix}, D = \begin{pmatrix}5\\2\end{pmatrix}, \qquad P = \begin{pmatrix}1\\2\end{pmatrix}, R = \begin{pmatrix}3\\1\end{pmatrix}, S = \begin{pmatrix}5\\4\end{pmatrix}, T = \begin{pmatrix}3\\5\end{pmatrix}.$$

5. Consider the clipping polygon $ABCDE$, given by its vertices

$$A = \begin{pmatrix}2\\2\end{pmatrix}, B = \begin{pmatrix}5\\1\end{pmatrix}, C = \begin{pmatrix}6\\3\end{pmatrix}, D = \begin{pmatrix}4\\5\end{pmatrix}, E = \begin{pmatrix}2\\4\end{pmatrix}.$$

Clip the lines PR and ST by the polygon $ABCDE$, if

$$P = \begin{pmatrix}1\\4\end{pmatrix}, R = \begin{pmatrix}1\\1\end{pmatrix}, S = \begin{pmatrix}4\\3\end{pmatrix}, T = \begin{pmatrix}7\\2\end{pmatrix},$$

using the algorithm *Cyrus-Beck*.

7 Rasterization

1. Rasterize the oriented line segment AB:

$$A = \begin{pmatrix} -3 \\ 2 \end{pmatrix}, \ B = \begin{pmatrix} -6 \\ 6 \end{pmatrix}$$

   using the Bresenham algorithm. Transform the line segment into the octant V., derive the rules for rasterization in this octant, rasterize the line segment in the octant V. and then transform it back to the original octant. Write the resulting coordinates of the drawn pixels in the raster.

2. Derive **in detail** the rules for rasterization of a line segment in the IInd octant, using the *Bresenham* algorithm.

   Now compute the coordinates of the drawn pixels of the line segment XY by

   - transforming it into the IInd octant,
   - rasterizing it in the IInd octant,
   - transforming it back to the original octant.

   The coordinates of the endpoints of XY are

$$X = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \ Y = \begin{pmatrix} 6 \\ 2 \end{pmatrix}.$$