BIKUBICKY STMEĽOVANÉ COONSOVE ZÁPLATY

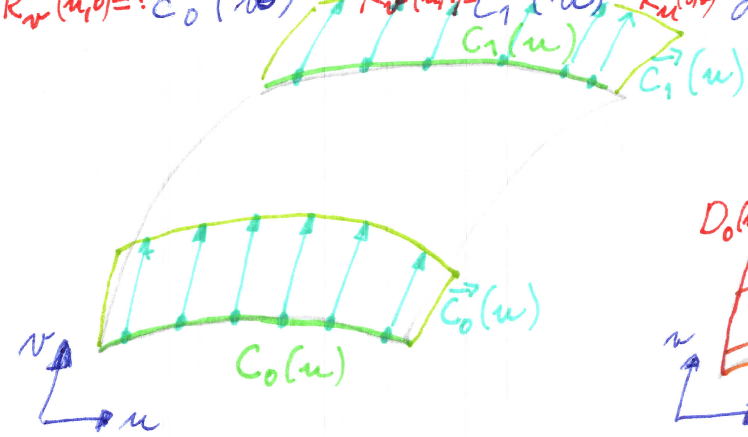
→ VSTUP: • štyri okrajové krivky

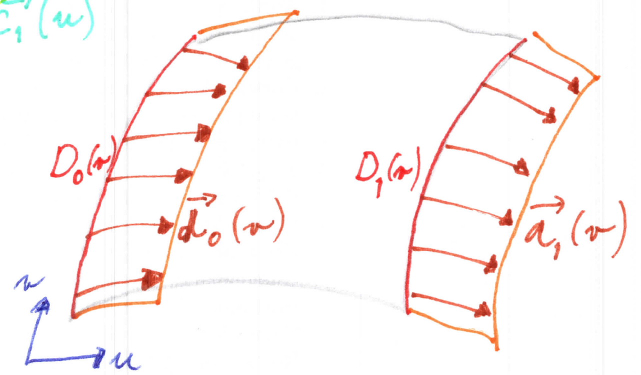
$R(u,0)=C_0(u)$, $C_1(u)$ [$R(u,1)$], $D_0(v)$ [$R(0,v)$], $D_1(v)=R(1,v)$

→ spĺňajú podmienku C^0-kompatibility

• štyri priečne derivácie:

$R_v(u,0)=:\vec{c}_0(u)$ $\quad R_v(u,1)=:\vec{c}_1(u)$ $\quad R_u(0,v)=:\vec{d}_0(v)$ $\quad \vec{d}_1(v)=R_u(1,v)$





• vektory twistov v rohových bodoch

$\vec{t}_{00}=R_{uv}(0,0)$ $\qquad \vec{t}_{01}=R_{uv}(0,1)$

$\vec{t}_{10}=R_{uv}(1,0)$ $\qquad \vec{t}_{11}=R_{uv}(1,1)$

→ VÝSTUP: záplata $R(u,v)=R_C(u,v)+R_D(u,v)-R_{CD}(u,v)$, kde

interpolant krivík $C_0(u)$ a $C_1(u)$ ← $R_C(u,v)=h_{03}(v)\,C_0(u)+h_{13}(v)\,\vec{c}_0(u)+h_{23}(v)\,\vec{c}_1(u)+h_{33}(v)\,C_1(u)$

$=h_{03}(v)\,R(u,0)+h_{13}(v)\,R_v(u,0)+h_{23}(v)\,R_v(u,1)+h_{33}(v)\,R(u,1)$

interp. krivík $D_0(v)$ a $D_1(v)$ ← $R_D(u,v)=h_{03}(u)\,D_0(v)+h_{13}(u)\,\vec{d}_0(v)+h_{23}(u)\,\vec{d}_1(v)+h_{33}(u)\,D_1(v)$

$=h_{03}(u)\,R(0,v)+h_{13}(u)\,R_u(0,v)+h_{23}(u)\,R_u(1,v)+h_{33}(u)\,R(1,v)$

$$R_{CD}(u,v)=\begin{pmatrix}h_{03}(u) & h_{13}(u) & h_{23}(u) & h_{33}(u)\end{pmatrix}\begin{pmatrix}R(0,0) & R_v(0,0) & R_v(0,1) & R(0,1)\\ R_u(0,0) & R_{uv}(0,0) & R_{uv}(0,1) & R_u(0,1)\\ R_u(1,0) & R_{uv}(1,0) & R_{uv}(1,1) & R_u(1,1)\\ R(1,0) & R_v(1,0) & R_v(1,1) & R(1,1)\end{pmatrix}\begin{pmatrix}h_{03}(v)\\ h_{13}(v)\\ h_{23}(v)\\ h_{33}(v)\end{pmatrix}$$

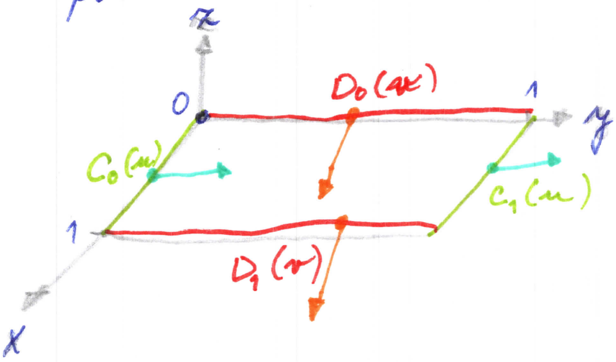
~~$R_{CD}(u,v) = (h_{03}(u)\ h_{13}(u)\ h_{23}(u)\ h_{33}(u)) \begin{pmatrix} C_0(0) \\ \\ \\ C_0(1) \end{pmatrix}$~~

$$h_{03}(t) = 1 - 3t^2 + 2t^3$$
$$h_{13}(t) = t - 2t^2 + t^3$$
$$h_{23}(t) = -t^2 + t^3$$
$$h_{33}(t) = 3t^2 - 2t^3$$

~~→ vidíme, že napr.~~

$$R_{CD}(u,v) = (h_{03}(u)\ h_{13}(u)\ h_{23}(u)\ h_{33}(u)) \begin{array}{c} \begin{array}{cccc} D_0(0) & & & D_0(1) \\ \parallel & & & \parallel \end{array} \\ \begin{pmatrix} C_0(0) & \vec{c_0}(0) & \vec{c_1}(0) & C_1(0) \\ \vec{d_0}(0) & \vec{t}_{00} & \vec{t}_{01} & \vec{d_0}(1) \\ \vec{d_1}(0) & \vec{t}_{10} & \vec{t}_{11} & \vec{d_1}(1) \\ C_0(1) & \vec{c_0}(1) & \vec{c_1}(1) & C_1(1) \end{pmatrix} \\ \begin{array}{cccc} \parallel & & & \parallel \\ D_1(0) & & & D_1(1) \end{array} \end{array} \begin{pmatrix} h_{03}(v) \\ h_{13}(v) \\ h_{23}(v) \\ h_{33}(v) \end{pmatrix}$$

→ príklad s rovinou: a $\vec{t}_{00} = \vec{t}_{10} = \vec{t}_{01} = \vec{t}_{11} = (0,0,T)$



$C_0(u) = (u,0,0)^T \quad u \in [0,1]$

$C_1(u) = (u,1,0)^T \quad u \in [0,1]$

$D_0(v) = (0,v,0)^T \quad v \in [0,1]$

$D_1(v) = (1,v,0)^T$

→ zvolme priečne derivácie tak, že sú to jednotkové vektory kolmé na krivky ležiace v rovine $z = 0$.

$\vec{c_0}(u) = (0,1,0)^T \qquad \vec{d_0}(v) = (1,0,0)^T$

$\vec{c_1}(u) = (0,1,0)^T \qquad \vec{d_1}(v) = (1,0,0)^T$

$$R_D(u,v) = (1-3u^2+2u^3 \quad u-2u^2+u^3 \quad -u^2+u^3 \quad 3u^2-2u^3) \begin{pmatrix} 0, v, 0 \\ 1, 0, 0 \\ 1, 0, 0 \\ 1, v, 0 \end{pmatrix} =$$

$$= \begin{pmatrix} u \\ v \\ 0 \end{pmatrix}$$

$$R_C(u,v) = (1-3v^2+2v^3 \quad v-2v^2+v^3 \quad -v^2+v^3 \quad 3v^2-2v^3) \begin{pmatrix} u, 0, 0 \\ 0, 1, 0 \\ 0, 1, 0 \\ u, 1, 0 \end{pmatrix} = \begin{pmatrix} u \\ v \\ 0 \end{pmatrix}$$

$$R_{CD}(u,v) = \left(1-3u^2+2u^3 \quad u-2u^2+u^3 \quad -u^2+u^3 \quad 3u^2-2u^3\right) \cdot \begin{pmatrix} (0,0,0) & (0,1,0) & (0,1,0) & (0,1,0) \\ (1,0,0) & (0,0,T) & (0,0,T) & (1,0,0) \\ (1,0,0) & (0,0,T) & (0,0,T) & (1,0,0) \\ (1,0,0) & (0,1,0) & (0,1,0) & (1,1,0 \end{pmatrix} \begin{pmatrix} \cdot \\ \cdot \\ \cdot \end{pmatrix}$$

$$= \left((u,0,0) \quad (0,1,T(u-3u^2+2u^3)) \quad (0,1,T(u-3u^2+2u^3)) \quad (u,1,0)\right) \begin{pmatrix} 1-3v^2+2v^3 \\ v-2v^2+v^3 \\ -v^2+v^3 \\ 3v^2-2v^3 \end{pmatrix} =$$

$$= \left(u,\ v,\ T(u-3u^2+2u^3)(v-3v^2+2v^3)\right)$$

$$R(u,v) = R_C + R_D - R_{CD} = \begin{pmatrix} u \\ v \\ 0 \end{pmatrix} + \begin{pmatrix} u \\ v \\ 0 \end{pmatrix} - \begin{pmatrix} u \\ v \\ T(u-3u^2+2u^3)(v-3v^2+2v^3) \end{pmatrix} =$$

$$= \begin{pmatrix} u \\ v \\ -T(u-3u^2+2u^3)(v-3v^2+2v^3) \end{pmatrix}$$