Representations of objects

Quadtree, octree and BSP
Meshes, half-edge and winged edge data structures

Marcel Makovník

Department of Algebra and Geometry
Faculty of Mathematics, Physics and Informatics
Comenius University in Bratislava, Slovakia

3 October 2019

Motivation, needs

- CAGD, displaying data, game design, entertainment
- different needs – different representations

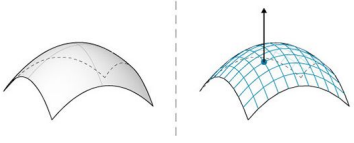


Figure: ”nice” image

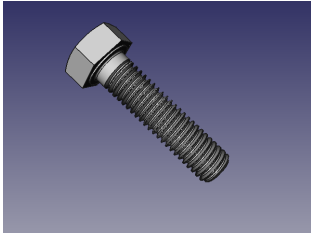


Figure: precise model

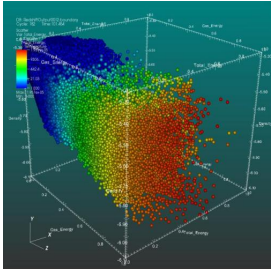
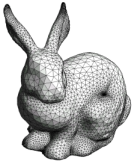
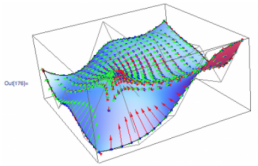


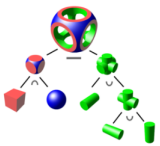
Figure: data visualization

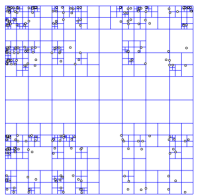
Types of object representations

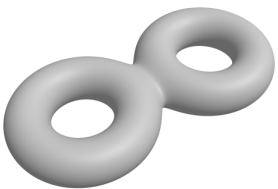
- meshes
- splines
- CSG
- space partitioning
- implicit surfaces
- etc.











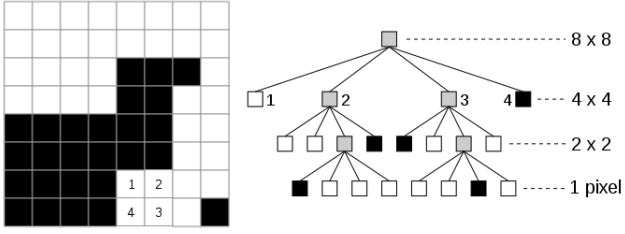
Space partitioning

- dividing (usually Euclidean) space into two or more disjoint subsets
- usage in ray tracing (wait for course CG(2))

- quadtree
- octtree
- BSP tree
- ~~k-d tree~~

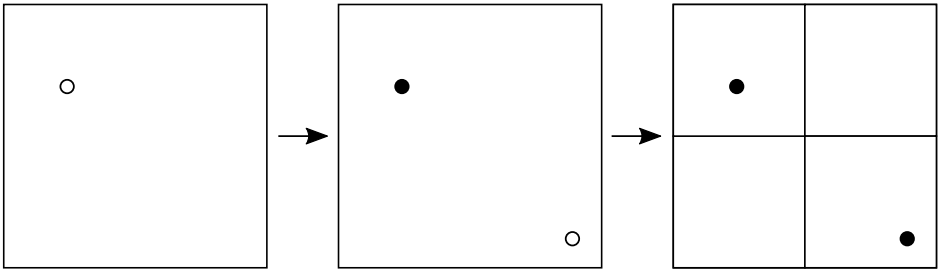
Quadtree – definition

- tree (data structure) with each branch having exactly 4 children



Quadtree – features

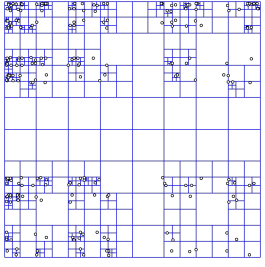
- decomposition of a space into adaptive cells (usually squares or rectangles)
- each cell has maximum capacity
- if the capacity has been exceeded, the cell is subdivided



Quadtree – types

- **region quadtree**
  - max. capacity – one (or more) colour in a cell
  - e.g. *image representation*

- **point quadtree**
  - max. capacity – one (or more) point in a cell
  - e.g. *mesh generation*

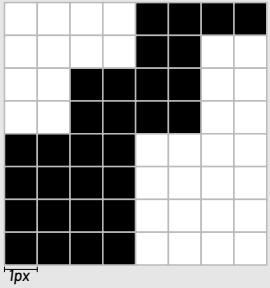


Quadtree – representations

- as a **tree** data structure
- as a **text string** (sometimes called *linear quadtree*)

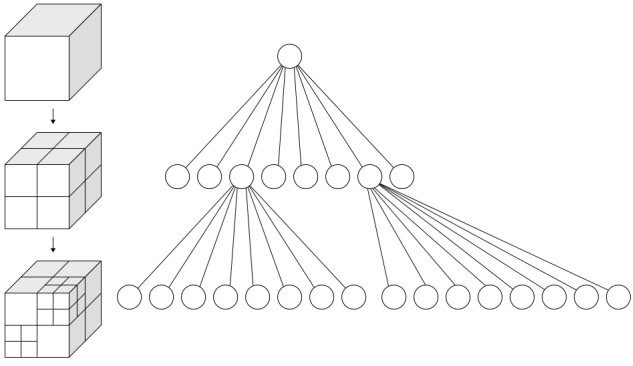
Exercise

Record the following monochromatic image using quadtree in both tree and line notation:



Octree

- each branch has exactly 8 children
- one subdivision step creates 8 octants
- 3D analogy of quadtree
- e.g. *collision detection in game design*



Binary space partitioning – BSP

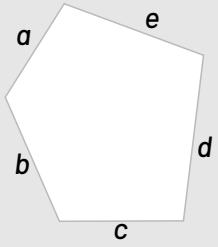
- binary tree
- one subdivision step divides a space into convex sets by **hyperplanes**
- generalization of both quadtree and octree
- store and retrieve spatial information about primitives
- static geometry
- e.g. *painter's algorithm, CSG - operations with shapes*

BSP – generation

- scene is divided into two until partitioning satisfies one or more criteria
- for points (???), for polygons (???)

Exercise

Generate BSP tree for a pentagon. What is the optimal hyperplane for the initial subdivision?



Vertex, edge and face

Definition

Let $\mathcal{V} = \{V_1, ..., V_n\},\ V_i \in \mathbb{R}^3,\ i = 1, ..., n$. Then V_i is called a ***vertex*** and $\mathcal{V}$ is called a ***set of vertices***.

Definition

Denote $\mathcal{E} \subseteq \mathcal{V}^{[2]}$, where $\mathcal{V}^{[2]} = \{[P_i, P_j] \in \mathcal{V}^2 \mid P_i \neq P_j\}$ and each $e = [V_i, V_j] \in \mathcal{E}$ satisfies $[V_j, V_i] = e$. Then e is called an ***edge*** and $\mathcal{E}$ is called a ***set of edges***.

Definition

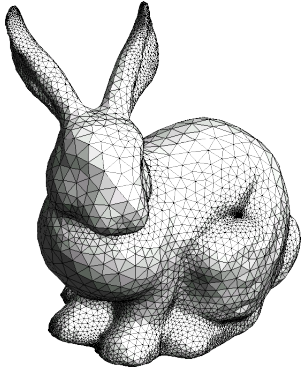
Let $\mathcal{F} \subseteq \mathcal{V}^{[n]},\ n \geq 3$, where $\mathcal{V}^{[n]} = \{[V_{i_1}, V_{i_2}, ..., V_{i_n}] \in \mathcal{V}^n \mid V_{i_j} \neq V_{i_k}, j \neq k\}$. The set $\mathcal{F}$ is called a ***set of faces***. Its element $\mathbf{F} = [V_{i_1}, V_{i_2}, ..., V_{i_n}] \in \mathcal{F}$ is called a ***face*** and conforms the condition, that all permutations of $V_{i_1}, V_{i_2}, ..., V_{i_n}$ represent the same face $\mathbf{F}$.

Mesh

Definition

Denote $\mathcal{M} = \{\mathcal{V}, \mathcal{E}, \mathcal{F}\}$. $\mathcal{M}$ is called a ***polygon mesh***. The corresponding sets of vertices, edges and faces are denoted by $\mathcal{V}(\mathcal{M})$, $\mathcal{E}(\mathcal{M})$ and $\mathcal{F}(\mathcal{M})$, respectively.

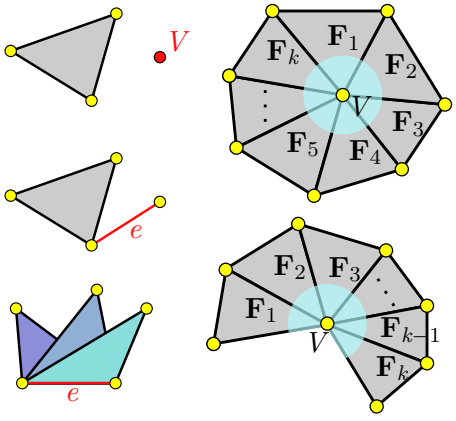
A mesh is meant to be a **2-manifold**, meaning each vertex on a mesh $\mathcal{M}$ has a neighbourhood that is homeomorphic to the Euclidean space $\mathbb{E}^2$.



Mesh

To avoid non-manifold meshes, additional conditions are required:

1. each vertex $V \in \mathcal{V}$ lies on some of the edges from $\mathcal{E}$
2. each edge $e \in \mathcal{E}$ lies on some of the faces from $\mathcal{F}$
3. each edge $e \in \mathcal{E}$ belongs to at most two faces from $\mathcal{F}$
4. for each face $[P_i, P_j, P_k] \in \mathcal{F}$ we have $[P_i, P_j], [P_i, P_k], [P_j, P_k] \in \mathcal{E}$,
5. for each vertex $V \in \mathcal{V}$ and all faces $\mathbf{F}_1, ..., \mathbf{F}_k \in \mathcal{F}$ sharing V, one of the following statements applies:
   - $\mathbf{F}_1, ..., \mathbf{F}_k$ create a simple cycle around V
   - $\mathbf{F}_1, ..., \mathbf{F}_k$ create a simple sequence around V



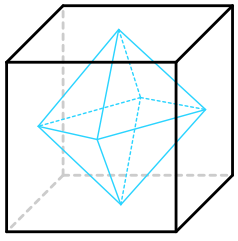
Mesh

Definition

Boundary of a mesh is the union of all edges belonging to exactly one face. The mesh is said to be **closed**, if it has no boundary.

Definition

The mesh is **connected** if the dual graph is connected.

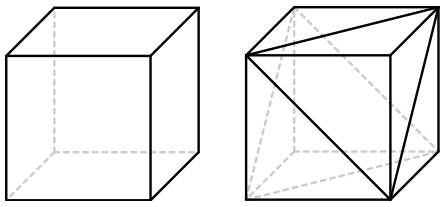


Polygon mesh – properties

- **Euler's formula**

$$\#V - \#E + \#F = 2(1 - g),$$

g – genus *(number of handles)*, $\#V, \#E, \#F$ – number of vertices, edges and faces, respectively



For triangle meshes:

- **number of vertices, faces and edges**

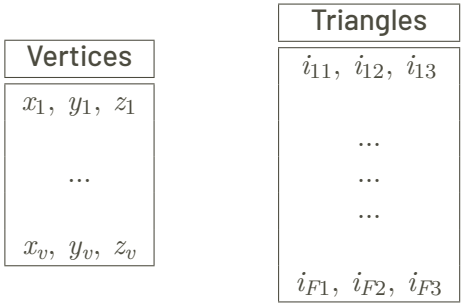
$$\#F \approx 2\#V, \quad \#E \approx 3\#V$$

- average **vertex valence** (number of incident edges) is 6

How to represent a mesh?

- the efficiency and memory consumption depends on the data structure
- we need to take into account
  - **topological requirements** – *2-manifold meshes/complex edges or singular vertices, triangle/polygon meshes etc.*
  - **algorithmic requirements** – *just render the mesh, local neighbourhoods of vertices/edges/faces, static/changing geometry etc.*

Face-based data structures

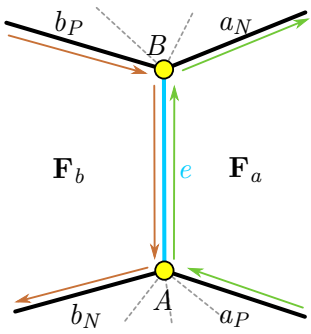


- vertices $\longleftrightarrow$ coordinates
- triangles $\longleftrightarrow$ indices

- static data
- formats (.obj, .off), VRML and OpenGL

- no information about additional connectivity
- traversal is expensive

Edge-based data structures (winged-edge)



Vertex
coinciding edge
coordinates

Faces
coinciding edges

Edges
coinciding vertices
coinciding faces
prev. and next edge of the left traverse
prev. and next edge of the right traverse

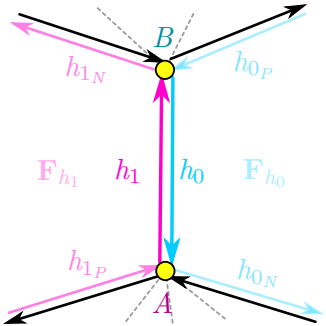
- connectivity via edges
- clockwise ordering
- fast traversal through edges, vertices and faces
- case distinctions

Halfedge-based data structure

Vertex
halfedge pointing to the vertex
coordinates

Faces
coinciding halfedge

Edges
vertex it points to
adjacent face
next halfedge
previous halfedge
opposite halfedge



- edge is split into two halfedges
- consistent counter-clockwise ordering
- only for 2-manifolds

- C++ library – **OpenMesh**