Problems to solve during the (out)break

PART 1

1. *Lambertian reflectance* says, that the light intensity on an ideally diffuse surface is directly proportional to cosine of the angle β, which is created by the normal vector of the surface $\mathbf{n}$ and the incoming light ray $\mathbf{s}$.

   Calculate the light intensity in the point $(0, 10, 0)^\top$, which lies on a surface with the normal vector $\mathbf{n} = (0, 1, 0)^\top$ and the coordinates of the light source are $(20, 20, 40)^\top$.

   Notes, additional questions and hints:

   - Ideally diffuse surfaces do not occur in real world.
   - The light intensity is computed as $\cos\beta = \mathbf{n} \cdot \mathbf{s}$.
   - When is the light intensity highest/lowest relative to the value of the angle β?
   - How to explain the situation where $\cos\beta < 0$?

   Solution: The light intensity is $1/\sqrt{21} \approx 0.218$.

2. Using the *Snell's law* $\eta_1 \sin\theta_1 = \eta_2 \sin\theta_2$, and the fact $\|\mathbf{s}\| = \|\mathbf{n}\| = 1$, prove that following equations for an ideally reflected and refracted ray hold:

$$\mathbf{s}_o = 2(\mathbf{s} \cdot \mathbf{n})\mathbf{n} - \mathbf{s}$$

$$\mathbf{s}_l = -\frac{\eta_1}{\eta_2}(\mathbf{s} - (\mathbf{s} \cdot \mathbf{n})\mathbf{n}) - \sqrt{1 - \left(\frac{\eta_1}{\eta_2}\right)^2 (1 - (\mathbf{s} \cdot \mathbf{n})^2)}\, \mathbf{n}$$

   Notes, additional questions and hints:

   - $\mathbf{s}$ – incoming ray (note the direction),
     $\mathbf{n}$ – normal vector,
     $\mathbf{s}_o$ – reflected ray,
     $\mathbf{s}_l$ – refracted ray,
     θ_1 – angle of reflection,
     θ_2 – angle of refraction,
     η_1, η_2 – indices of refraction (IOR) for different media (each material has unique IOR).

   - Snell's law may be applied only to isotropic materials.
     - *isotropic materials* – the angle of reflection (the visual appearance) does not change under the rotation around the normal. The most of the common materials are isotropic, e. g. *glass, air*.

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  - *anisotropic materials* – the angle of reflection changes under the rotation around the normal. The examples of anisotropic materials are *silk* or *wood*.

3. Air has the refraction index equal to 1.00026 and for water it is 1.33. Consider the ray passing from the water into the air. Compute the critical angle for the water, i.e. when the total internal reflections occurs.

   Notes, additional questions and hints:

   - The total internal reflection (broadly said, there is no refraction) occurs only if the light travels from the medium with higher IOR to the medium with lower IOR.
   - The critical angle is dependent on the feasible values of the sine of the respective angle.
   - What happens if the angle of reflection is greater than the critical angle?

   Solution: The critical angle is $\approx 48.77°$.

4. Consider a light ray which strikes the glass plate (with constant non-zero thickness) placed in the vacuum. We know, that the velocity of light distribution in the glass is 54% of the one for vacuum.

   What is the angle of refraction when the ray strikes the glass plate at an angle 60°? What is the angle of refraction, when the ray leaves the glass plate?

   Notes, additional questions and hints: The ratio of the velocities of light distribution is inversely proportional to the ratio of respective refractive indices.

   Solution: The angle of refraction is $\approx 27.882°$ and $60°$, respectively.